# LPI AND BER PERFORMANCE OF A CHAOTIC CDMA SYSTEM USING DIFFERENT DETECTION STRUCTURES

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#### ABSTRACT

Low probability of intercept (LPI) performance of a direct-sequence code division multiple access (DS-CDMA) system is investigated in this paper; both chaotic and pseudorandom binary spreading sequences are considered. Several intercept receiver structures, including energy detector, synchronous and asynchronous, coherent and noncoherent, are examined, and the expressions of the detection probabilities are derived. The bit error rate (BER) of the chaotic CDMA system is also investigated in the paper.

### 1 INTRODUCTION

Covert operation is required for a transmitter to protect the radio signals so that a commercial or military interceptor has difficulty in detecting the presence of the radio signals. Spread spectrum modulation can be used in a radio system to reduce the likelihood of intercept, as well as providing protection against jamming and interference [1]. Direct sequence spread spectrum is one of the spread spectrum techniques. Traditionally, a pseudorandom (PN) sequence is used for direct-sequence code division multiple access (DS-CDMA) systems, but it lacks security due to fact that there are limited number of available PN sequences and they show periodic correlation properties. Studies in nonlinear dynamical systems have developed chaotic theories. Chaotic sequences, based on chaotic theories, are nonbinary and nonperiodic sequences. The number of available chaotic sequences for a DS-CDMA systems can be very large. It is very difficult for an interceptor to decipher the chaotic sequence even if a chaotic function is known. The properties of chaotic sequences provide advantages over the conventional PN sequences based systems.

The detection probability of the presence of transmitted waveforms from a single user using a PN sequence has been investigated with different kinds of interceptors/detectors, such as energy detectors and optimum intercept receivers [2–4]. Chaotic spreading sequences have been proposed to be used in DS-CDMA systems to improve the low probability of intercept (LPI) performance [5–8]. LPI performance of a chaotic signal has been studied in [6]. Up to now however, no work has studied the performance of a chaotic CDMA system (with multiple users). In this paper, we investigate the LPI and bit error rate (BER) performance of a DS-CDMA system using chaotic sequences. Synchronous communication environment (downlink) over additive white gaussian noise (AWGN) channel is considered. Both PN binary spreading sequences and chaotic spreading sequences are examined.

## 2 CHAOTIC CDMA SYSTEM MODELS

### 2.1 Chaotic Sequences

Binary PN sequences are used in conventional DS-CDMA systems. The most commonly used PN sequence is m-sequence which is generated by a linear feedback shift register and the state-machine go through each register state by a deterministic manner. The m-sequence is a periodic sequence and its length is determined by the number of shift registers. There are very limited number of m-

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Form Approved OMB No. 0704-0188 sequences for a given shift register code generator. By using code clock extraction techniques, an intercepter can wipe out the spreading sequences and leave out the unspreaded modulated user information. In order to improve the covertness of the communications, noise-like chaotic spreading sequences can be used to conceal the signals. Several chaotic maps, such as logistic map, triangular map, and exponential map can be used to generate chaotic sequences [9–11]. The logistic map is one of the simplest and most widely studied. The chaotic sequences using logistic map can be sequentially generated by the following equation

$$x_{n+1} = \alpha(1 - x_n) \tag{1}$$

where  $0 \leq x_n \leq 1$ ,  $0 \leq \alpha \leq 4$ , and  $\alpha$  is called bifurcation parameter. The generated sequences can change dramatically depending one the value of  $\alpha$ . For  $0 \leq \alpha \leq 3.57$ , the sequence  $x_n$  is periodic with a period  $2^m$  for some integer m; while for  $3.57 \leq \alpha \leq 4$ , the sequence is nonperiodic and nonconverging [6,9-11]. In this paper, we use  $\alpha=4$  for logistic map and different chaotic sequences are generated by different initial states  $x_0$ . The probability density function (pdf) for some chaotic sequences has been derived. For example, the sequences generated by triangular map are uniform over the internal [0,1]. The pdf of he sequences based on logistic map is the following

$$f(x_n) = \frac{1}{\pi \sqrt{x_n(1 - x_n)}} \tag{2}$$

In order to change the chaotic sequence into bipolar signal which is suitable for spread spectrum, the following transform is taken,  $a_n = 2x_n - 1$ . The corresponding pdf of  $a_n$  is

$$f(a_n) = \frac{1}{\pi \sqrt{(1 - a_n^2)}}$$
 (3)

The sequence  $a_n$  is used as the chaotic DS-CDMA spreading sequence throughout this paper.

#### 2.2 Chaotic CDMA System

Considering the downlink of a chaotic DS-CDMA system, all the transmitted signals from different users are synchronized at the base station. If all the K users are assumed to have same power the transmitted signal can be expressed as follows

$$s(t) = \sum_{k=1}^{K} \sqrt{2P} a_k(t) b_k(t) \cos(\omega_c t + \phi) \qquad (4)$$

in which  $a_k$  is the spreading sequence and the  $b_k$  is the information for the kth user. P is chosen

such that  $PE[\alpha^2(t)]$  is the average signal power,  $\omega_c = 2\pi f_c$  is the carrier frequency and  $\phi$  is the phase. The spreading sequence can be expressed as

$$a_k(t) = \sum_{n = -\infty}^{\infty} a_k(n) p(t - nT_c - \epsilon T_c)$$
 (5)

in which  $T_c$  is the chip period and p(t) is a unitamplitude pulse of duration  $T_c$  seconds. The chip epoch  $\epsilon T_c$  can be modeled by a random variable  $\epsilon$ , uniformly distributed in [0,1). The intended receiver knows the spreading sequence, and it can recover the information  $b_k(t)$  by multiplying the  $a_k(t)$  to the received signal. Since the chaotic spreading sequence  $a_k(t)$  is very difficult for intercepter to get, the information  $b_k(t)$  is concealed.

#### 2.3 Detection Schemes

Based on the incident waveforms, an intercept receiver must decide between the signal-plus-noise  $(H_1)$  and the noise-only hypothesis  $(H_0)$ 

$$r(t) = \begin{cases} \sum_{k=1}^{K} \sqrt{2P} a_k(t) \cos(\omega_c t + \phi) + n(t) & H_1 \\ n(t) & H_0 \end{cases}$$
(6)

where n(t) is bandpass AWGN with one-sided power spectral density of  $N_0$  W/Hz. The observation time is T seconds, which is assumed to be an integer multiple of chip duration.  $T = NT_c$  and N is positive integer. There is a strong assumption embedded in (6), namely, under hypothesis  $H_1$ , the signals from all the users are present and all the user information bit  $b_k(t) = 1$  during the whole observation interval. This eliminates the possibility of the signal either starting of ending during the observation time and the case that some users are transmitting different information bit in [0, T]. Such an assumption is important because it is the case that the group communication can be detected more easily by interceptors. In another word, it is the worst case for the covert CDMA system. It also provides meaningful and fairly simple comparative conclusions which can be extended to more general models.

# 3 LPI PERFORMANCE OF A CHAOTIC CDMA SYSTEM

The LPI performance of a chaotic CDMA system is studied in this section. The configurations of intercept detectors depend on the amount of known features of the signals. For example, an energy detector only assumes that the signals occupy a bandwidth of W and exist for a time duration T. Other intercept receivers could use the known features of

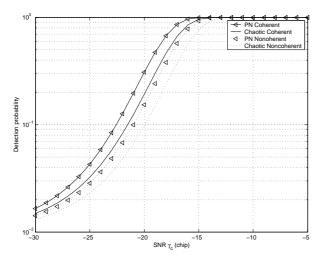


Figure 1: LPI performance of synchronous intercept receiver with  $P_{FA} = 0.01$ , N = 1000, and the number of users K = 4.

the spread spectrum signals, such as the carrier frequencies, chip rates, and  $T=NT_c$ , where N is the number of chips in one observation. In the optimum detection for binary sequences, a receiver implements a likelihood ratio test (LRT), which is a procedure based on statistical signal testing of hypotheses [2], [4], [5]. In the following, we evaluate five intercept receivers for chaotic signal detection, considering coherent (known  $\phi$ ) and non-coherent, synchronous ( $\epsilon=0$ ) and asynchronous detections, and energy detections.

# 3.1 Synchronous Coherent Intercept Receiver

When synchronous coherent intercept receivers are used to detect the presence of chaotic signals, both the chip epoch  $\epsilon T_c$  and the carrier phase  $\phi$  are assumed to be known by the intercepter, and we have  $z(t) = r(t)\cos(\omega_0 t + \phi)$  at the intercepter side. For binary sequence detection, a likelihood ratio test is developed in [4]. For non-binary sequence detection (using binary correlation), following [6], we are able to establish that the decision between  $H_0$  and  $H_1$  can be made based on the rule below

$$\Lambda(z(t)) = \prod_{j=1}^{N} \exp(-\frac{PT_c}{N_0}) \cosh(\frac{2\sqrt{P}}{N_0}r_j) \underset{H_0}{\overset{H_1}{\gtrless}} \Lambda_0$$

where  $\Lambda_0$  is a threshold and  $r_j = \int_{(j-1)T_c}^{jT_c} z(t)dt$ . For typical chip signal-to-noise ratio (SNR) below -5 dB, we have an approximated expression for log-

LRT[4]

$$\lambda = \sum_{j=1}^{N} r_j^2 \underset{H_0}{\overset{H_1}{\geqslant}} \lambda_0 \tag{8}$$

For large values of N,  $\lambda$  can be approximated as Gaussian for both noise alone and signal-plus-noise cases and this approximation is quite accurate [2], [4]. Therefore the detection probability,  $P_D$ , can be determined via a Q function. Chaotic sequences for different users are assumed to have perfect correlation

$$E[a_i(t)a_j(t)] = 0 (9)$$

For the random variable  $\lambda$ , we are able to find its mean and variance

$$\begin{cases} m_{\lambda} = N(N_0 T_c)(0.5 + \gamma_c K C \delta_{k,1}) \\ \sigma_{\lambda}^2 = N(N_0 T_c)^2 [0.5 + (2K C \gamma_c + K D \gamma_c^2) \delta_{k,1}] \end{cases}$$

where  $C = \mathrm{E}[a_n^2]/\mathrm{E}[|a_n|], D = \mathrm{Var}[a_n^2]/\mathrm{E}^2[|a_n|],$  $\delta_{k,1}$  is the Kronecker delta function, which implies the presence of a signal  $(H_1)$  for k = 1 or the absence of the signal  $(H_0)$  for k = 0, and the chip SNR

$$\gamma_c = \frac{PT_c \mathbf{E}[|a_n|]}{N_0} \tag{10}$$

Notice that in deriving or implementing (10), we assume that a binary correlating function (Sign( $\sum_{k=1}^{K} a_k(t)$ )) is used to detect a non-binary chaotic sequence ( $\sum_{k=1}^{K} a_k(t)$ ). Therefore, the detectors for binary spreading sequences are applied directly to detect the presence of chaotic spreading signals. This approach is proposed in [6] to simplify the receiver structure. Otherwise, an optimum chaotic receiver structure, which requires the exact correlating functions matching to the received sequences, is not feasible because of the infinite sequence combinations of the non-binary chaotic signals. In deriving the detection probability, we first find a threshold level  $\lambda_0$  by setting an acceptable false alarm probability  $P_{FA}$ . Using the obtained  $\lambda_0$ , the detection probability  $P_D$  is derived as a function of  $P_{FA}$ 

$$P_{D} = Q \left( \frac{Q^{-1}(P_{FA}) - \sqrt{2N}\gamma_{c}KC}{\sqrt{1 + 4KC\gamma_{c} + 2KD\gamma_{c}^{2}}} \right)$$
(11)

where  $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp(-t^2/2) dt$ . It is also interesting to note that this binary correlating approach and its related performance analysis are applicable to any DS-CDMA system with multilevel spreading sequences.

# **3.2 Synchronous Noncoherent Intercept** found in [6] as Receiver

We relax the assumption the carrier phase of the chaotic CDMA waveforms is know by the intercepter. In this case, the carrier phase  $\phi$  is modeled as a random variable with a uniform distribution in  $[0,2\pi)$ . Match filters followed by envelope detectors are used to combine with the binary correlation of noncoherent detection of chaotic CDMA signals. This receiver structure is proposed in [6] and the decision rule is as (8) with

$$r_j = \sqrt{r_{I_j}^2 + r_{Q_j}^2}$$

where

$$\left[\begin{array}{c} r_{I_j} \\ r_{Q_j} \end{array}\right] = \sqrt{2} \int_{jT_c}^{(j+1)T_c} r(t) \left[\begin{array}{c} \cos \omega_c t \\ \sin \omega_c t \end{array}\right] dt$$

and j=0,...,N-1. The mean and variance of the decision statistics  $\lambda$  is

$$\begin{cases} m_{\lambda} = N(N_0 T_c)(1 + \gamma_c K C \delta_{k,1}) \\ \sigma_{\lambda}^2 = N(N_0 T_c)^2 [1 + (2K C \gamma_c + 0.5K D \gamma_c^2) \delta_{k,1}] \end{cases}$$

The detection probability of synchronous noncoherent chaotic CDMA signals is obtained as

$$P_D = Q \left( \frac{Q^{-1}(P_{FA}) - \sqrt{N}\gamma_c KC}{\sqrt{1 + 2KC\gamma_c + 0.5KD\gamma_c^2}} \right)$$
(12)

# 3.3 Asynchronous Coherent Intercept Receiver

In most cases, the chip timing (epoch)  $\epsilon$  is unknown by the intercepter. It can be modeled as a random variable uniformly distributed in  $[0,T_c)$ . Asynchronous coherent intercept receiver is investigated in [4]. Two epoches values  $\epsilon=0$  and  $\epsilon=0.5T_c$  are assumed to get the analytical results. In this paper, we first derive a conditional detection probability for a given chip epoch  $\epsilon$ . Then the final detection probability is derived by averaging the conditional detection probability over all possible chip epoch  $\epsilon$  values. The detection decision variables can be

$$\lambda = \sum_{j=0}^{N-1} \left\{ PT_c^2 \left[ \epsilon^2 \left( \sum_{k=1}^K \alpha_k(j) - \sum_{k=1}^K \alpha_k(j+1) \right)^2 + 2 \left( \sum_{k=1}^K \alpha_k(j) - \sum_{k=1}^K \alpha_k(j+1) \sum_{k=1}^K \alpha_k(j+1) \epsilon \right) + \left( \sum_{k=1}^K \alpha_k(j+1) \right)^2 + n_I^2 + 2\sqrt{P} n_I T_c \times \left( \sum_{k=1}^K \alpha_k(j) \epsilon + \sum_{k=1}^K \alpha_k(j+1) (1-\epsilon) \right) \right\}$$
(13)

where  $n_I = \sqrt{2} \int_0^{T_c} n(t) \cos 2\pi f_0 t dt$ . The mean and variance of  $\lambda$  are derived as follows

$$\begin{cases} m_{\lambda} = N(N_0 T_c)(0.5 + \gamma_c KC(1 - 2\epsilon + 2\epsilon^2)\delta_{k,1}) \\ \sigma_{\lambda}^2 \approx N(N_0 T_c)^2 [0.5 + 2KC\gamma_c (1 - 2\epsilon + 2\epsilon^2)\delta_{k,1}] \end{cases}$$

and the detection probability conditioned on  $\epsilon$  is

$$P_{D|\epsilon} = Q \left( \frac{Q^{-1}(P_{FA}) - \sqrt{2N}(1 - 2\epsilon + 2\epsilon^2)\gamma_c KC}{\sqrt{1 + 4KC\gamma_c(1 - 2\epsilon + 2\epsilon^2)}} \right)$$
(14)

If  $\epsilon$  is assumed be uniformly distributed in [0,1), the average detection probability of the chaotic CDMA system using an asynchronous coherent intercept receiver is

$$\overline{P_D} = \int_0^1 Q \left( \frac{Q^{-1}(P_{FA}) - \sqrt{2N}(1 - 2\epsilon + 2\epsilon^2)\gamma_c KC}{\sqrt{1 + 4KC\gamma_c(1 - 2\epsilon + 2\epsilon^2)}} \right) d\epsilon$$
(15)

### 3.4 Asynchronous Noncoherent Intercept Receiver

The most natural problem formulation is that the chip epoch  $\epsilon$  and carrier phase  $\phi$  are random variables. Followed by (12) in [6], the decision statistics

can be derived as

$$\lambda = PT_c^2 \sum_{j=0}^{N-1} \left[ \epsilon^2 \left( \sum_{k=1}^K \alpha_k(j) - \sum_{k=1}^K \alpha_k(j+1) \right)^2 + \left( \sum_{k=1}^K \alpha_k(j) \right)^2 \right]$$

$$+ 2\sqrt{P}T_c \epsilon \sum_{j=0}^{N-1} \left( \sum_{k=1}^K \alpha_k(j) - \sum_{k=1}^K \alpha_k(j+1) \right) \times \left( T_c \sqrt{P} \sum_{k=1}^K \alpha_k(j+1) + N(n_I \cos \phi + n_Q \sin \phi) \right)$$

$$+ 2\sqrt{P}T_c \sum_{j=0}^{N-1} \left[ \left( \sum_{k=1}^K \alpha_k(j+1) \right) \times (n_I \cos \phi + n_Q \sin \phi) \right] + N(n_I^2 + n_Q^2)$$

$$\times (n_I \cos \phi + n_Q \sin \phi) + N(n_I^2 + n_Q^2)$$

$$(16)$$

where  $n_Q = \sqrt{2} \int_0^{T_c} n(t) \sin 2\pi f_0 t dt$ . We then derived the mean and variance of  $\lambda$  as

$$\begin{cases} m_{\lambda} = N(N_0 T_c)(1 + \gamma_c KC(1 - 2\epsilon + 2\epsilon^2)\delta_{k,1}) \\ \sigma_{\lambda}^2 \approx N(N_0 T_c)^2 [1 + 2KC\gamma_c(1 - 2\epsilon + 2\epsilon^2)\delta_{k,1}] \end{cases}$$

The detection probability of the chaotic CDMA signals conditioned on  $\epsilon$  is found as

$$P_{D|\epsilon} = Q \left( \frac{Q^{-1}(P_{FA}) - \sqrt{N}(1 - 2\epsilon + 2\epsilon^2)\gamma_c KC}{\sqrt{1 + 2KC\gamma_c(1 - 2\epsilon + 2\epsilon^2)}} \right)$$
(17)

and the average detection probability using asynchronous noncoherent receiver is

$$\overline{P_D} = \int_0^1 Q \left( \frac{Q^{-1}(P_{FA}) - \sqrt{N}(1 - 2\epsilon + 2\epsilon^2)\gamma_c KC}{\sqrt{1 + 2KC\gamma_c(1 - 2\epsilon + 2\epsilon^2)}} \right) d\epsilon$$
(18)

### 3.5 Energy Detector

Energy detector is one of the simplest detectors whose block diagram can be found in [2] and [3]. It consits of a bandpass filter, a square-law device, a finite-time integrator, a sampler that samples the integrator output at the end of the integration interval T, and a threshold comparison device [2]. The decision statistics is the sampler output which can be written as

$$V = \frac{2}{N_0} \int_0^T r^2(t)dt = \frac{2}{N_0} \sum_{j=0}^{N-1} \int_{iT_c}^{(i+1)T_c} r^2(t)dt$$

When the integrator time-bandwidth product WT is large, V is also approximated as Gaussian, and

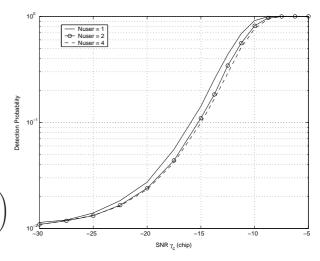


Figure 2: LPI performance of a chaotic CDMA system with different number of users.  $P_{FA} = 0.01$  and N = 1000.

this approximation is quite accurate [2], [3]. Following the derivation in [6] we get the mean and variance of the decision statistics as

$$\begin{split} m_V &= 2WT \left( 1 + \frac{KP \mathbf{E}[\alpha_n^2]}{N_0 W} \delta_{k,1} \right) \\ \sigma_V^2 &= 2WT + 4KPT \left( \frac{PT \mathbf{Var}[\alpha_n^2]}{N_0^2} + 2 \frac{\mathbf{E}[\alpha_n^2]}{N_0} \right) \delta_{k,1} \end{split}$$

The detection probability is

$$P_{D} = Q \left( \frac{Q^{-1}(P_{FA}) - \sqrt{\frac{N}{2}}K\gamma_{c}}{\sqrt{1 + KC\gamma_{c} + \frac{NHK\gamma_{c}^{2}}{2}}} \right)$$
(20)

where  $\gamma_c = PT_c E[\alpha_n^2]/N_0$  and  $H = Var[\alpha_n^2]/E^2[\alpha_n^2]$ .

# 4 BER PERFORMANCE OF A CHAOTIC CDMA SYSTEM

BER performance of CDMA systems using PN sequences has been extensively studied. Standard Gaussian approximation (SGA), improved Gaussian approximation (IGA), and simplified IGA (SIGA) have been used to model the interference statistics [12]. In this section, we study the downlink performance of a chaotic CDMA system which all the signals are transmitted with the same chip epoch. The received signal at downlink of a chaotic CDMA system can be written as

$$r(t) = \sum_{k=1}^{K} \sqrt{2P} a_k(t) b_k(t) \cos(\omega_c t + \phi) + n(t)$$
 (21)

Without loss the generality, we study the performance of the first intended receiver. It knows the

spreading sequence  $\alpha_1(t)$ , carrier frequency  $\omega_c$  and phase  $\phi$ . The decision statistics at the reference receiver after demodulation and despreading is

$$Z_{1} = \sqrt{P} \left( \sum_{n=1}^{G} \alpha_{1}^{2}(n) \right) b_{1}(t)$$

$$+ \sum_{k=2}^{K} \sqrt{P} \left( \sum_{n=1}^{G} a_{k}(n) a_{1}(n) \right) b_{k}(t) + n_{1}$$
(22)

where G is the processing gain and  $n_1 = \int_0^{GT_c} \sqrt{2}n(t)\cos(\omega_c t + \phi)dt$  is AWGN with one-sided power spectral density of  $GN_0$  W/Hz and  $b_k(t)$  is the binary bit information from the kth user. If autocorrelation function of chaotic sequences with length G is defined as

$$R_i(l) = \frac{1}{G} \sum_{n=1}^{G} a_i(n) a_i(n+l)$$
 (23)

and cross-correlation between sequence  $\alpha_k$  and  $\alpha_j$  is defined as

$$R_{k,j}(l) = \frac{1}{G} \sum_{n=1}^{G} a_k(n) a_j(n+l)$$
 (24)

then the decision statistics can be written as

$$Z_1 = G\sqrt{P}R_1(0)b_1(t) + \sum_{k=2}^K G\sqrt{P}R_{k,1}(0)b_k(t) + n_1$$

If  $b_1(t) = 1$  the error occurs only is  $Z_1 < 0$ . For large number of K,  $Z_1$  can be modeled as a Gaussian random variable. Its mean and variance is

$$\begin{cases} m_{Z_1} = G\sqrt{P}T_c \mathbb{E}[\alpha_1^2] \\ \sigma_{Z_1}^2 = GPT_c^2 \text{Var}[\alpha_1^2] + GKPT_c^2 \mathbb{E}^2[\alpha_1^2] + GN_0 W \end{cases}$$

The BER can be calculated as

$$P_e = Q \left( \frac{\sqrt{G\gamma_c} E[\alpha_1^2]}{\sqrt{\gamma_c Var[\alpha_1^2] + K\gamma_c E^2[\alpha_1^2] + 1}} \right)$$
 (26)

where the chip SNR  $\gamma_c = \frac{PT_c}{N_0}$  and the above result is based on SGA method.

#### 5 NUMERICAL RESULTS

Numerical results of the performance for a chaotic CDMA system with logistic map is presented in this section. The logistic map chaotic sequences have C=0.79, D=0.32, and H=0.8. Random binary sequences have C=1, D=0, and H=0. The LPI performance of binary a CDMA system and a

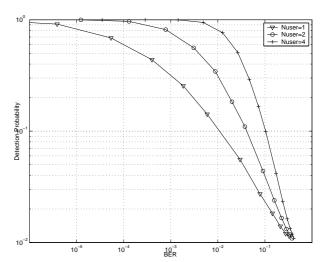


Figure 3: LPI performance vs. BER performance of a chaotic CDMA system.  $P_{FA} = 0.01$ , N = 1000, and processing gain G = 100.

chaotic CDMA system is compared in Fig. 1. Better LPI performance is observed for chaotic CDMA system. The chip SNR improvement is about 1 dB.Detection probability of a chaotic CDMA system with different number of users is plotted in Fig. 2. Total power of 1, 2, and 4 users is assumed to be same. It shows that the LPI performance is improved with increased number of users. In another word, if the total power of chaotic CDMA system is constant, the more users are communicating in the system the more difficult an intercepter will detect its presence. Fig. 3 plots the LPI performance versus BER performance of a chaotic CDMA system. Cases of different number of users with the same individual power are compared. The processing gain G = 100 is assumed. It is seen that for the same BER level the more users the system supports the worse LPI performance the system has. It means chaotic CDMA systems have to sacrifice LPI performance to gain better communication quality. It is also true for traditional CDMA systems.

### 6 CONCLUSIONS

The performance of a chaotic CDMA system is studied in this paper. Detection probability is evaluated based on five different intercept receiver structures, including synchronous/asynchronous and coherent/noncoherent receivers and energy detector. Analytical results shows chaotic CDMA systems have better LPI performance than traditional binary PN spreading CDMA systems. It is seen from numerical results that the chaotic CDMA system will be less likely to be detected if it increases

the number of users while maintaining the same total power. BER performance of a chaotic CDMA system is also investigated in this paper and it is seen that the system has to sacrifice its communication quality to maintain its LPI performance.

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